

Cosmological Neutrino Condensates

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Abstract

We investigate the possibility that neutrinos form superfluid-type condensates in background cosmological densities. Such condensates could give rise to small neutrino masses and splittings, as well as an important contribution, perhaps, to the cosmological constant. We discuss various channels in the context of the standard model. Many of these do not support a condensate, but some mixed-flavor channels do. We also suggest a new interaction, acting only among neutrinos, that could induce a neutrino Majorana mass of order 1 eV.

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For various reasons, but especially the recent evidence for neutrino oscillations and hence neutrino mass [1], considerable effort has been devoted of late to neutrinos and their properties. Furthermore, another more speculative consideration has piqued interest in small neutrino masses: a number of cosmological observations has led to renewed interest in the long-discarded notion of a cosmological constant [2]. The preferred value of this constant leads to a mass scale that is quite similar to the mass for neutrinos inferred from the atmospheric and solar neutrino observations, in the range of tenths of an eV to a few eV [1]. It seems to us that a natural framework in which to relate these seemingly disparate phenomena would be the formation of some type of neutrino condensate, the energy of which might provide the cosmological constant, while the neutrino masses could emerge from an expansion about the symmetry-breaking vacuum.

Appropriate tools to investigate the possibility of neutrino pair condensation of the superconductor type, have been recently employed to explore the possibility of qq condensates in hadronic media at sufficiently high densities. This has been discussed in the context of a four-fermi approximation to QCD , whether induced by instanton effects [3] or by one-gluon exchange [4]. Similar effects have also been studied in the large- N approximation in a 2-dimensional model related to the Gross-Neveu model [5].

For our purposes, we look for pairing phenomena in the electroweak theory. Specifically, we suggest that condensates involving neutrinos may occur in cosmological situations where the relevant chemical potentials are non-zero. Of course, the chemical potentials of the various neutrinos in the universe are presumably quite tiny [6], reflecting as they do the difference in density between neutrinos and anti-neutrinos of a given species. We shall return to the estimation of the magnitude of the effects we discuss below; first we offer a purely theoretical discussion of the following problem: to identify, if any, the attractive channels in the electroweak theory that might permit neutrinos to condense. It should be noted that the earliest speculation on this subject of which we are aware [7], took place before the emergence of the standard model and the discovery of neutral currents, and the authors simply assumed the existence of an attractive channel.

The hallmark of this type of pairing is that, once a Fermi surface forms, any attractive channel, no matter how weak, can produce a condensate; this is in contrast to the more familiar situation in hadronic physics of a chiral condensate, which typically requires the coupling to exceed a certain threshold value.

The reason for this behavior of the pairing condensate can be understood in a variety of ways. If one looks at the gap equation in a mean-field approximation one sees that, as the gap tends to zero, a singularity develops at the Fermi surface; hence a non-zero gap is

necessary to prevent the formation of this singularity.

Alternatively, in a renormalization-group approach [8] one sees that the renormalized coupling tends to infinity as one integrates out all modes above and below the Fermi surface; thus even an apparently weak coupling grows without bound and allows the condensate to form.

Our investigation will be limited to the leptonic sector of the electroweak theory. We shall use the effective four-Fermi low-energy description of that theory and we shall perform all our computations within the one loop (mean-field) approximation. Corrections to this picture from the inclusion of effects not captured by the low-energy effective theory, or from fluctuations about the mean-field approximation, must await further investigations. We shall discuss possible consequences of extensions of the dynamics beyond the standard model below.

To set the stage, let us consider an action with a generic four Fermi coupling:

$$S = \frac{1}{2}[\psi_\alpha^\dagger A_{\alpha\beta} \psi_\beta - \psi_\alpha A_{\alpha\beta}^T \psi_\beta^\dagger] + \mathcal{M}_{\alpha\beta\gamma\delta} \psi_\alpha^\dagger \psi_\beta \psi_\gamma^\dagger \psi_\delta .$$

Here the index α summarizes all the attributes of ψ : space-time dependence, gauge transformation properties, flavor, spin and whatever else.

Because we are interested in pairings of the form $\langle\psi\psi\rangle$ and $\langle\psi^\dagger\psi^\dagger\rangle$, we assume that \mathcal{M} has a Fierz-Bogoliubov (FB) decomposition:

$$\mathcal{M}_{\alpha\beta\gamma\delta} = \sum_\lambda \eta_\lambda Q_{\alpha\gamma}^{(\lambda)} Q_{\beta\delta}^{*(\lambda)} .$$

Here $\eta_\lambda = \pm 1$, and $Q_{\alpha\gamma}^{(\lambda)} = -Q_{\gamma\alpha}^{(\lambda)}$ because of the Fermi statistics of ψ . Then we define auxiliary fields $B^{(\lambda)}$, and add to \mathcal{L} the term

$$- \sum_\lambda \eta_\lambda (B^{(\lambda)\dagger} - Q_{\alpha\gamma}^{(\lambda)} \psi_\alpha^\dagger \psi_\gamma^\dagger) (B^{(\lambda)} + Q_{\beta\delta}^{*(\lambda)} \psi_\beta \psi_\delta) .$$

The original Lagrangian is recovered upon path-integrating on $B^{(\lambda)}$ and $B^{(\lambda)\dagger}$. With the addition of this extra piece, the terms in \mathcal{L} that are quartic in ψ and ψ^\dagger cancel, leaving us with

$$\begin{aligned}
S &= \frac{1}{2}[\psi^\dagger A \psi - \psi A^T \psi^\dagger] - \sum_\lambda \eta_\lambda B^{\dagger(\lambda)} B^{(\lambda)} \\
&\quad + \psi^\dagger \mathcal{B} \psi^\dagger + \psi \mathcal{B}^\dagger \psi , \\
\mathcal{B}_{\alpha\gamma} &= \sum_\lambda \eta_\lambda B^{(\lambda)} Q_{\alpha\gamma}^{(\lambda)} , \\
\mathcal{B}_{\beta\delta}^\dagger &= - \sum_\lambda \eta_\lambda B^{\dagger(\lambda)} Q_{\beta\delta}^{*(\lambda)} .
\end{aligned}$$

We obtain an effective action $\Gamma_{eff}[B^{(\lambda)}, B^{(\lambda)\dagger}]$ by integrating over ψ and ψ^\dagger . The result is:

$$\Gamma_{eff} = - \sum_\lambda \eta_\lambda B^{(\lambda)\dagger} B^{(\lambda)} - \frac{i}{2} \text{Tr} \log[\mathbf{1} + 4A^{-1} \mathcal{B} (A^T)^{-1} \mathcal{B}^\dagger]$$

where we have dropped some terms independent of B and B^\dagger . The mean field approximation that we shall adopt consists in demanding that Γ be stationary with respect to variations of B and B^\dagger :

$$\frac{\delta \Gamma}{\delta B^{(\lambda)}} = \frac{\delta \Gamma}{\delta B^{(\lambda)\dagger}} = 0 .$$

These are the gap equations whose non-trivial solutions, if any, determine whether a condensate with the quantum numbers of $B^{(\lambda)}$ and $B^{(\lambda)\dagger}$ can form. Because the effective theory we are dealing with is non-renormalizable, it is necessary to regularize these gap equations, and the size of the gap depends, apparently, on the value of the cutoff. Thus the gap equations in the form we derive them are not a good guide to the size of the gap. However, they allow us to determine which channels are attractive and hence in which channels one might expect a condensate to form. We shall see explicitly how this works in particular cases below.

The simplest case is only one flavor of neutrino interacting with itself via the neutral current. Since vector exchange produces repulsion among like particles, we intuitively expect the $\nu - \nu$ channel to be repulsive in this case. This is borne out by our explicit computation, which we now describe.

The starting point is the four-Fermi interaction

$$\mathcal{L}_{int} = -G^2 \bar{\psi} \gamma_\mu \psi \bar{\psi} \gamma^\mu \psi$$

where $\gamma_5 \psi = -\psi$. The sign is dictated by the standard model. If we use 2-component fermions, and make use of the identity $\sum_a \sigma_{\alpha\beta}^a \sigma_{\gamma\delta}^a = 2[\epsilon_{\alpha\gamma} \epsilon_{\delta\beta} + \frac{1}{2} \delta_{\alpha\beta} \delta_{\gamma\delta}]$ we see that

$$\mathcal{L}_{int} = 2G^2(\psi_\alpha^\dagger \epsilon_{\alpha\gamma} \psi_\gamma^\dagger)(\psi_\beta \epsilon_{\beta\delta} \psi_\delta) . \quad (1)$$

Making contact with our previous notation, we see that there is only one term in the FB decomposition with $Q_{\alpha\gamma} = \sqrt{2}G\epsilon_{\alpha\gamma}$, and $\eta = -1$. We remark that the fact that $\eta = -1$ already tells us that the interaction is repulsive at the tree level; it is then to be expected that the gap equation will not have a non-trivial solution.

For the kinetic part of the action, again using 2-component notation, we have $A = i\frac{\partial}{\partial t} - i\vec{\sigma} \cdot \vec{\nabla} - \mu$, where μ is the chemical potential. Hence

$$A^{-1} = \int \frac{d^4p}{(2\pi)^4} \left[\frac{(p_0 - \mu) - \vec{p} \cdot \vec{\sigma}}{(p_0 - \mu + i\epsilon \text{sgn} p_0)^2 - \vec{p}^2} \right] e^{-ip \cdot (x-y)}$$

and

$$(A^{-1})^T = - \int \frac{d^4p}{(2\pi)^4} \frac{[(p_0 + \mu) - \vec{p} \cdot \vec{\sigma}^T]}{(p_0 + \mu + i\epsilon \text{sgn} p_0)^2 - \vec{p}^2} e^{-ip \cdot (x-y)} .$$

Note that the $i\epsilon$ prescription has been introduced in the appropriate manner to take account of the role of μ as the chemical potential. One then proceeds to construct $X \equiv 4A^{-1}\mathcal{B}(A^T)^{-1}\mathcal{B}^\dagger$ which, under the assumption (appropriate for a vacuum solution) that B and B^\dagger are constants, can be written

$$X = - \int \frac{d^4p}{(2\pi)^4} \mathcal{F}(p) e^{-ip \cdot (x-y)}$$

where

$$\mathcal{F}(p) = \frac{+8G^2 B^\dagger B}{p_0^2 - (\vec{p} \cdot \vec{\sigma} - \mu)^2 + i\epsilon} .$$

The gap equation will involve $\text{Tr}(1 + X)^{-1}X$. Doing the p_0 integral in this expression provides a factor of i that cancels the explicit i appearing in Γ_{eff} . The remaining integral over \vec{p} is ultraviolet divergent. We regularize this by imposing a cutoff Λ on the magnitude of \vec{p} . This leads to the unrenormalized gap equation, written in terms of $M = B^\dagger B$

$$1 = \frac{-G^2}{\pi^2} \int_{-\Lambda}^{\Lambda} dp p^2 \frac{1}{\sqrt{(p - \mu)^2 + 8MG^2}} .$$

Clearly there is no solution (whereas there would have been a solution if we had had $\eta = 1$; this would have changed the sign of the l-h s). Note that for $M = 0$, there is a logarithmic divergence coming from $p = \mu$. This is the singularity at the Fermi surface mentioned above.

Including an arbitrary number N of flavors turns out to be fairly straightforward assuming no flavor mixing. One begins with the analog of eqn. (1):

$$\mathcal{L}_{int} = 2G^2(\psi_\alpha^{\dagger(i)}\epsilon_{\alpha\gamma}\psi_\gamma^{\dagger(j)})(\psi_\beta^{(i)}\epsilon_{\beta\delta}\psi_\delta^{(j)}) \quad (2)$$

where i, j are flavor indices summed from 1 to N , and performs a separate FB transformation on the flavor indices using the identity

$$2\delta_{ik}\delta_{jl} = \frac{2}{N}\delta_{ij}\delta_{kl} + \sum_{a=1}^{N^2-1}(\lambda^a)_{ij}(\lambda^a)^*_{kl}$$

where the λ 's are the generators of the fundamental representation of $SU(N)$, normalized such that $Tr\lambda^a\lambda^b = 2\delta^{ab}$.

Following an analysis paralleling the one-flavor case,[‡] one finds

$$X_{\alpha\beta}^{ij}(x-y) = -M_{ij} \int \frac{d^4p}{(2\pi)^4} \mathcal{F}_{\alpha\beta}(p) e^{-ip \cdot (x-y)}$$

where

$$\mathcal{F} = \frac{4G^2}{p_0^2 - (\vec{p} \cdot \vec{\sigma} - \mu)^2 + i\epsilon}$$

and

$$M_{ij} = \sum_{A,B} B^{(A)} B^{(B)\dagger} \lambda_{ik}^{(A)} \lambda_{kj}^{(B)} .$$

Here the summation runs over the symmetric λ 's, including the unit matrix suitably normalized. It is important to note that M is a positive matrix: $M = KK^\dagger$, $K = \sum_A B^{(A)} \lambda^{(A)}$.

This expression for X leads to the following set of gap equations:

$$B^{(A)} = \frac{-G^2}{2\pi^2} B^{(B)} \int_{-\Lambda}^{\Lambda} dp p^2 tr \left[\frac{1}{\sqrt{(p-\mu)^2 + 4G^2 M}} \lambda^{(B)} \lambda^{(A)} \right] .$$

Multiply by $B^{(A)\dagger}$ and sum on A . This produces

[‡]In this analysis, we assume a common chemical potential for all the flavors. What may happen if this is not the case will be discussed briefly below.

$$\sum_A B^{(A)\dagger} B^{(A)} = \frac{-G^2}{2\pi^2} \int_{-\Lambda}^{\Lambda} dp p^2 \text{tr} \left[\frac{1}{\sqrt{(p-\mu)^2 + 4G^2 M}} M \right]$$

whose only solution is $B^{(A)} = 0$ for all A .

Thus, if we confine ourselves to neutrinos alone, and to the dynamics of the standard model, we find no possibility of neutrino pairing. Among the ways to avoid this conclusion are (a) extend the dynamics beyond the standard model (we shall discuss this possibility in the conclusions); (b) enlarge the dynamics to include the charged leptons (and possibly also the quarks). We have performed an analysis in which we consider not only neutrinos themselves but also electrons circulating in the loop. This generates an additional term in Γ_{eff} , and hence an additional contribution to the gap equation for the neutrino condensate. It does not, however, alter the result that there is no solution to the gap equation; (c) Finally, we can consider condensates that are composed not of neutrinos alone, but that pair neutrinos with charged leptons. Of course, since these condensates would be charged, their phenomenological consequences would be much more drastic than those of purely neutrino condensates.

In any event, we begin with the Lagrange density $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$, where

$$\begin{aligned} \mathcal{L}_0 &= \bar{e}(i \not{\partial} - m)e + \bar{\nu}_e i \not{\partial} \nu_e + \bar{\nu}_\mu i \not{\partial} \nu_\mu - \mu_e e^\dagger e \\ &\quad - \mu_\nu \nu_e^\dagger \nu_e - \mu'_\nu \nu_\mu^\dagger \nu_\mu . \\ \mathcal{L}_{int} &= \frac{-g^2}{8m_W^2} [\bar{\nu}_e \gamma_\mu (1 - \gamma_5) e \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e - \frac{1}{2} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) \nu_e \bar{e} \gamma^\mu (1 - \gamma_5) e \\ &\quad + 2 \sin^2 \theta_W \bar{e} \gamma^\mu e \bar{\nu}_e \gamma_\mu (1 - \gamma_5) \nu_e - \frac{1}{2} \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \nu_\mu \bar{e} \gamma^\mu (1 - \gamma_5) e \\ &\quad + 2 \sin^2 \theta_W \bar{e} \gamma^\mu e \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \nu_\mu] . \end{aligned}$$

We have kept only the electron, as by far the lightest charged lepton. We have exhibited its couplings to ν_e and ν_μ ; there should also be ν_τ , which we ignore for simplicity. It enters in an identical manner to ν_μ .

Now we must implement the FB transformation. It is convenient first to make a Fierz transformation on the charged-current terms, to get them in a form resembling the neutral current terms, and then to make a FB transformation on the result. We find

$$\begin{aligned} \mathcal{L}_{int} &= \frac{g^2}{8m_W^2} [4 \sin^2 \theta_W (\bar{\nu}_e \gamma_\mu \bar{e} \nu_e \gamma^\mu e + \bar{\nu}_\mu \gamma_\mu \bar{e} \nu_\mu \gamma^\mu e) \\ &\quad + 4(1 + 2 \sin^2 \theta_W) \bar{\nu}_e \gamma^0 \gamma^2 \bar{e} \nu_e \gamma^0 \gamma^2 e \\ &\quad + 4(-1 + 2 \sin^2 \theta_W) \bar{\nu}_\mu \gamma^0 \gamma^2 \bar{e} \nu_\mu \gamma^0 \gamma^2 e] . \end{aligned}$$

Note that $\gamma^0\gamma^2$ is the charge conjugation matrix, and hence expressions of the form $\nu\gamma^0\gamma^2e$ are proper Lorentz scalars. If we assume that the condensate will be a Lorentz scalar (this is only for convenience: there is no particular reason to expect that these condensates should respect Lorentz symmetry) then we limit ourselves to the last 2 terms. Note that they are of opposite sign. It turns out that the term involving ν_e is "bad" ($\eta = -1$) whereas the one involving ν_μ is "good" ($\eta = +1$). Hence we concentrate only on the last term, and introduce an auxiliary field B for it. We let $\kappa^2 = \frac{g^2}{2M_W^2}(1 - 2\sin^2\theta_W)$, and for notational convenience we introduce a doublet $\psi = (e, \nu_\mu)$ (a peculiar object from the point of view of the underlying standard model), so that, for example, $m\bar{e}e = \frac{m}{2}\bar{\psi}(1 + \tau_3)\psi$ and $\mu_e e^\dagger e + \mu'_\nu \nu_\mu^\dagger \nu_\mu = \psi^\dagger(\mu_1 + \mu_2\tau_3)\psi$. In our earlier notation, we have $A = \gamma^0(i\not{X} - \frac{m}{2}(1 + \tau_3)) - \mu_1 - \mu_2\tau_3$ and $\mathcal{B} = -\frac{\kappa^2}{4}B(1 - \gamma_5)\gamma^2\gamma^0\tau_1$.

Following the same analytic path as before, we arrive at a somewhat more complicated gap equation:

$$-\kappa^2 B^\dagger B = -\frac{i\kappa^4 B^\dagger B}{\pi^3} \int_{-\infty}^{\infty} dp \, p^2 \int_{-\infty}^{\infty} dp_0 \mathcal{G}(p_0, p)$$

where

$$\mathcal{G}(p_0, p) = \frac{p_0 - \mu_+ + p}{[(p_0 - \mu_+)^2 - p^2 - m^2][p_0 + \mu_- + p] - 4\kappa^4 B^\dagger B[p_0 - \mu_+ + p]} .$$

Here $\mu_\pm = \mu_1 \pm \mu_2$, and the integration over poles on the real p_0 axis is by the prescription $p_0 \rightarrow p_0 + i\epsilon \text{sgn} p_0$.

It is not possible to make much analytic headway with the expression in the general case. But if we set $\Delta^2 = 4\kappa^4 B^\dagger B = 0$, the expression becomes tractable, and we can then address two questions: (i) does the integral have the correct sign to permit a solution of the gap equation? and (ii) is there an infrared singularity, which would be evidence of an instability that could be cured by setting $\Delta^2 \neq 0$? Let $\omega = \sqrt{p^2 + m^2} > 0$. Then the p_0 integral can be done, yielding

$$\begin{aligned} 1 = & \frac{\kappa^2}{\pi^2} \int_{-\Lambda}^{\Lambda} dp \, p^2 \left\{ \frac{\theta(\omega^2 - \mu_+^2)}{\omega} \left[\frac{(\omega - p)\theta(-p - \mu_-)}{\omega - p - 2\mu_1} + \frac{(\omega + p)\theta(p + \mu_-)}{\omega + p + 2\mu_1} \right] \right. \\ & \left. + \frac{4\mu_1}{(\omega + p + 2\mu_1)(\omega - p - 2\mu_1)} [\theta(-\mu_+ - \omega)\theta(-p - \mu_-) - \theta(\mu_+ + \omega)\theta(p + \mu_-)] \right\} . \end{aligned}$$

Because of the θ -functions, each term in the integrand is non-negative, thereby answering the first equation in the affirmative.

Further, one sees that singularities can occur only at the Fermi momentum, $p_F^2 = \mu_-^2$, and then only if the condition $\mu_+^2 - m^2 = \mu_-^2$ is met. Since the electron density is proportional to $[\mu_+^2 - m^2]^{3/2}$ while the neutrino density is proportional to μ_-^3 [9], this amounts to equating the Fermi momenta of the members of the pair. Phenomenologically, therefore, it is unlikely that this type of condensate could occur, except maybe in the early universe when larger background densities of both electrons and neutrinos were present.

We turn now to a consideration of the size of the condensate. A simple *BCS*-like estimate gives

$$\Delta \sim p_F e^{-\frac{1}{v_F^2 G^2}}.$$

If the G^2 in eq. (1) or (2) is of order G_F , then, given the allowed range of p_F , the exponential suppression makes Δ very small. One is led, therefore, to suggest the existence of a new interaction, acting only on neutrinos, for which the effective G^2 would have a scale of approximately 1 eV instead of the 200 GeV characteristic of G_F . An interaction of the same form as eqs. (1) or (2) would serve nicely, provided only we change the sign, thereby generating an attractive channel. The condensate would then produce Majorana neutrino masses of the form $m_\nu \sim \Delta = G^2 \langle \nu\nu \rangle$, and possibly a contribution to the cosmological constant $\Lambda \sim G^2 | \langle \nu\nu \rangle |^2$. Since both G and the fermi momentum p_F are of order 1 eV, one obtains neutrino masses and a cosmological constant that are likewise of this order.

Furthermore, if we generalize our earlier analysis and allow the chemical potentials for the different neutrino species to vary, the condensates could depend non-trivially on flavor, perhaps leading to an interesting spectrum of neutrino masses and mixings.

Our conclusions can be enumerated as follows:

- (i) There is no attractive channel in the purely neutrino sector of the standard model;
- (ii) The addition of charged leptons leads to attraction in the flavor off-diagonal channels, but a pairing instability occurs only if the Fermi momenta of the neutrino and the charged leptons are equal;
- (iii) In the case of neutrino-charged lepton pairing, there may also be the possibility of condensation in a Lorentz non-invariant channel. We have not looked at this in detail;
- (iv) If a new interaction exists among neutrinos with characteristic scale 1 eV, neutrino condensates could form with the right size to generate an interesting spectrum of masses and mixings, as well as an appropriate contribution to the cosmological constant. This possibility is currently under active investigation.

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REFERENCES

- [1] For reviews see E. Torrente-Lujan, hep-ph/9902339, and B. Kayser, hep-ph/9810513, and references therein.
- [2] S. Perlmutter, et al., astro-ph/9812473 and astro-ph/9812133; B. Schmidt, et al., *Astrophys. J.* **507**, 46 (1998); A.G. Riess, et al., astro-ph/9805200; Krauss, L.M. and Turner, M.S., *Gen. Rel. Grav.* **27**, 1137 (1995); Ostriker, J.P. and Steinhardt, P., *Nature* **377**, 600 (1995).
- [3] M. Alford, K. Rajagopal and F. Wilczek, *Phys. Lett.* **B422**, 247 (1998); R. Rapp, T. Shafer, E.V. Shuryak and M. Velkovski, *Phys. Rev. Lett.* **81**, 53 (1998); T. Schafer, *Nucl. Phys.* **A462**, 45 (1998); J. Berges and K. Rajagopal, *Nucl. Phys.* **B538**, 215 (1999).
- [4] M. Alford, K. Rajagopal and F. Wilczek, *Nucl. Phys.* **B537**, 443 (1999); R. Pisarski and D. Rischke, nucl-th/9811104; D.T. Son, hep-ph/9812287.
- [5] A. Chodos, H. Minakata and F. Cooper, *Phys. Lett. B* (to be published), hep-ph/9812305; and manuscript in preparation.
- [6] P.B. Pal and K. Kar, hep-ph/9809410.
- [7] V.L. Ginzburg and G.F. Zharkov, *ZhETF Piśma* **5**, 275 (1967) [*JETP Letters* **5**, 223 (1967)]; see also V. Alonso, J. Chela-Flores and R. Paredes, *Nuov. Cim.* **67B**, 213 (1982).
- [8] N. Evans, S.D.H. Hsu and M. Schwetz, hep-ph/9808444 and hep-ph/9810515; T. Shafer and F. Wilczek, hep-ph/9810509; D.T. Son (ref. 4); R. Shankar, *Rev. Mod. Phys.* **66**, 129 (1993); J. Polchinski, *TASI Lectures* (1992) (hep-th/9210046).
- [9] See for example, A. Chodos, K. Everding and D.A. Owen, *Phys. Rev.* **D42**, 2881 (1990), sec 2.